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GENERALIZATION OF GRADIENT METHOD OF HEAT TRANSFER INVESTIGATIONS FOR THE CASE OF ARBITRARY DEPENDENCE OF THE THERMAL CONDUCTIVITY COEFFICIENT OF THE WALL ON TEMPERATURE

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GENERALIZATION OF GRADIENT METHOD OF HEAT TRANSFER INVESTIGATIONS FOR THE CASE OF ARBITRARY DEPENDENCE OF THE THERMAL CONDUCTIVITY COEFFICIENT OF THE WALL ON TEMPERATURE

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ABSTRACT. The thermal fluxes on the inner surface of a tube, determined by the temperature distribution on its surfaces on the basis of a unidimensional temperature field, is compared for the purpose of evaluating the possible magnitude of error involved in the generalized gradient method using the analytical solution of Dirichlet's problem of the temperature field in a wall that transfers heat.

In the gradient method for investigating heat transfer, the thermal flux /105* is determined on the basis of the temperature gradient in the wall on the heat transfer surface, which is found, in turn, on the basis of the analytical solution of the Dirichlet problem of the temperature field in a wall that participates in heat transfer. For finding the analytical functions that express the temperature gradient on the heat transfer surface through the functions that reflect the temperature distribution on the boundaries of the wall, the thermal conductivity coefficient is assumed to be independent of temperature. The error associated with this simplification depends on the magnitude of the thermal stress, and when the heat transfer process is of low intensity, it can be disregarded. At the same time, in various branches of technology, particularly in rocketry, the importance of high-rate heat exchange processes is continuously increasing. In investigations by the gradient method of high-rate heat transfer processes, which occur in chemically reacting gas flows, in boiling liquids, and under other conditions, the prerequisite that the thermal conductivity coefficient be independent of temperature can result in large errors during processing of the experimental results.

In order to evaluate the possible magnitude of the error we will compare the thermal fluxes on the internal surface of a tube, found by the temperature

*Numbers in the margin indicate pagination in the foreign text.

distribution on its surfaces on the basis of a unidimensional temperature field. The solution of the differential equation of thermal conductivity with consideration of the linear dependence of the thermal conductivity coefficient on temperature

$$\lambda = \lambda_0 (1 + bt) \tag{1}$$

produces the following expression for thermal load per unit length of the tube

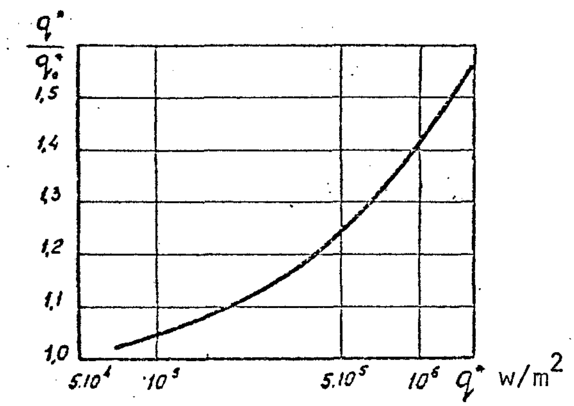
$$q^* = \frac{2\pi\lambda_0}{\ln \frac{d_2}{d_1}} \left[1 + \frac{b}{2}(t_1 + t_2)\right](t_1 - t_2), \tag{2}$$

where d_1 and d_2 are the internal and external diameters of the tube,
t_1 and t_2 are the temperatures of the surfaces.

By comparing this expression with the formula for the thermal load q_0^* /106
found with the assumption that the thermal conductivity coefficient is independent of temperature, we obtain [1]:

$$\frac{q^*}{q_0^*} = \frac{1}{2}\left(1 + \frac{1 + bt_2}{1 + bt_1}\right) = \frac{1}{2}\left(1 + \frac{\lambda_2}{\lambda_1}\right). \tag{3}$$

The graph shows q^*/q_0^* as a function of the thermal load q in terms of the area of the cylindrical surface of a tube for the average diameter, when the tube is made of EI 607 steel with a wall thickness of 10 mm and internal surface temperature of 100°C. The temperature of the outer surface under the maximum thermal stress reached 800°C. Information concerning the thermal conductivity coefficient of the steel was taken from [2].



The figure shows that when $q > 5 \cdot 10^4$ w/m^2, the processing of the experimental data without consideration of the dependence of λ on temperature for the examined conditions can result in considerable errors. At the same time, for instance, in chambers and nozzles of rocket engines, the thermal stresses can exceed substantially 10^6 w/m^2.

The use of the functions for unidimensional temperature fields in the gradient method for analyzing heat transfer enables us to solve only a narrow circle of problems. The possibilities of the method are greatly expanded by using analytical solutions for two-dimensional temperature fields. For a tube in axial flow and for transverse flow around the tube, and also for flow around a curvilinear wall, such solutions are examined in [3]-[5] in the case of arbitrary temperature distribution through the cross section of the wall.

For liquid flow in a tube, the average heat transfer coefficient through length z is determined by the formula [3]

$$\alpha = -\frac{\lambda_1}{\Delta t_{av} z}\,\varepsilon(z), \tag{4}$$

where

$$\varepsilon(z) = \int_0^z \left(\frac{\partial t}{\partial r}\right)_{r=r_1} dz. \tag{5}$$

and λ_1 is the thermal conductivity coefficient on the inner surface of the tube. /107

The function $\varepsilon(z)$ is found on the basis of the solution of the differential equation of thermal conductivity found with the assumption that the thermal conductivity coefficient is independent of temperature,

$$\frac{\partial^2 t}{\partial r^2} + \frac{1}{r}\frac{\partial t}{\partial r} + \frac{\partial^2 t}{\partial z^2} = 0 \tag{6}$$

for boundary conditions

$$\begin{aligned} r &= r_1 \quad t = \varphi(z), \\ r &= r_2 \quad t = \psi(z), \\ z &= 0 \quad t = \xi(r), \\ z &= l \quad t = \zeta(r). \end{aligned} \tag{7}$$

The solution has the form

$$\varepsilon(z)=\sum_{n=1}^{\infty}\left\{\left[B_n K_1\left(\frac{n\pi}{l}r_1\right)-A_n I_1\left(\frac{n\pi}{l}r_1\right)\right]\left(\cos\frac{n\pi}{l}z-1\right)-\right.$$
$$\left.-\left[C_n \operatorname{sh}\beta_n z+D_n(\operatorname{ch}\beta_n z-1)\right]V_1(\beta_n r_1)\right\}. \tag{8}$$

Here I_1 and K_1 are modified Bessel functions of the first order; A_n, B_n, C_n and D_n are constants found from the boundary condition; $V_1(\beta_n r_1)$ is a function; β_n are the eigenvalues of the problem.

The differential equation of thermal conductivity in the case of arbitrary dependence of the thermal conductivity coefficient on temperature can be represented in the Cartesian coordinate system as follows:

$$\frac{\partial}{\partial x}\left(\lambda\frac{\partial t}{\partial x}\right)+\frac{\partial}{\partial y}\left(\lambda\frac{\partial t}{\partial y}\right)+\frac{\partial}{\partial z}\left(\lambda\frac{\partial t}{\partial z}\right)=0. \tag{9}$$

Using auxiliary function [6]

$$G(t)=\frac{1}{\lambda_1}\int_0^t \lambda(t)\,dt \tag{10}$$

equation (9) can be transformed to

$$\frac{\partial^2 G}{\partial x^2}+\frac{\partial^2 G}{\partial y^2}+\frac{\partial^2 G}{\partial z^2}=0. \tag{11}$$

For the two-dimensional in cylindrical coordinates it acquires the same form as equation (6):

$$\frac{\partial^2 G}{\partial r^2}+\frac{1}{r}\frac{\partial G}{\partial r}+\frac{\partial^2 G}{\partial z^2}=0. \tag{12}$$

The boundary conditions for the problem are

$$\begin{aligned} r=r_1 &\quad G=\varphi^*(z), \\ r=r_2 &\quad G=\psi^*(z), \\ z=0 &\quad G=\xi^*(r), \\ z=l &\quad G=\zeta^*(r). \end{aligned} \tag{13}$$

Here ϕ^*, ψ^*, ξ^*, ζ^* are functions that characterize the distribution of G on the investigated wall, found with consideration of (7) and (10).

From (10) we obtain

$$\lambda_1 dG(r,\ z)=\lambda(t)\,dt(r,\ z). \tag{14}$$

Consequently /108

$$\lambda_1 \frac{\partial G}{\partial r}=\lambda(t)\frac{\partial t}{\partial r} \tag{15}$$

and

$$\lambda_1\left(\frac{\partial G}{\partial r}\right)_{r=r_1}=\lambda\,[\varphi(z)]\left(\frac{\partial t}{\partial r}\right)_{r=r_1}. \tag{16}$$

For the conditions under examination

$$\alpha=-\frac{1}{\Delta t_{\text{av}} z}\int_0^z \lambda[\varphi(z)]\left(\frac{\partial t}{\partial r}\right)_{r=r_1} dz. \tag{17}$$

With consideration of (16)

$$\alpha=-\frac{\lambda_1}{\Delta t_{\text{av}} z}\,\varepsilon^*(z), \tag{18}$$

where

$$\varepsilon^*(z)=\int_0^z\left(\frac{\partial G}{\partial r}\right)_{r=r_1} dz. \tag{19}$$

The function $\varepsilon^*(z)$ is determined by the solution of equation (12) for boundary conditions (13). Since (12) and (13) are of the same form as (6) and

(7), $\varepsilon^*(z)$ and $\varepsilon(z)$ are written analogously:

$$\varepsilon^*(z)=\sum_{n=1}^{\infty}\Big\{\Big[B_n^*K_1\Big(\frac{n\pi}{l}r_1\Big)-A_n^*I_1\Big(\frac{n\pi}{l}r_1\Big)\Big]\Big(\cos\frac{n\pi}{l}z-1\Big)-$$
$$-[C_n^*\operatorname{sh}\beta_n z+D_n^*(\operatorname{ch}\beta_n z-1)]V_1(\beta_n r_1)\Big\}. \tag{20}$$

The values $V_1(\beta_n r_1)$ and β_n are found in the same manner as for formula (8) [3].

We will find the coefficients that go into equation (20), assuming that the thermal conductivity coefficient depends linearly on temperature:

$$\lambda=\lambda_0+b^*t. \tag{21}$$

The function G(t) with consideration of (21) is rewritten in the form

$$G(t)=\frac{\lambda_0}{\lambda_1}t+\frac{b^*}{2\lambda_1}t^2. \tag{22}$$

Consequently

$$\begin{aligned}\varphi^*(z)&=\frac{\lambda_0}{\lambda_1}\varphi(z)+\frac{b^*}{2\lambda_1}\varphi^2(z),\\ \psi^*(z)&=\frac{\lambda_0}{\lambda_1}\psi(z)+\frac{b^*}{2\lambda_1}\psi^2(z),\\ \xi^*(r)&=\frac{\lambda_0}{\lambda_1}\xi(r)+\frac{b^*}{2\lambda_1}\xi^2(r),\\ \zeta^*(r)&=\frac{\lambda_0}{\lambda_1}\zeta(r)+\frac{b^*}{2\lambda_1}\zeta^2(r).\end{aligned} \tag{23}$$

We calculate the coefficients M_n^* and N_n^*, upon which depend A_n^* and B_n^* [3]: /109

$$M_n^*=\frac{2}{l}\int_0^l\varphi^*(z)\sin\frac{n\pi}{l}z\,dz=$$
$$=\frac{2}{l}\frac{\lambda_0}{\lambda_1}\int_0^l\varphi(z)\sin\frac{n\pi}{l}z\,dz+\frac{b^*}{\lambda_1 l}\int_0^l\varphi^2(z)\sin\frac{n\pi}{l}z\,dz,$$
$$N_n^*=\frac{2}{l}\int_0^l\psi^*(z)\sin\frac{n\pi}{l}z\,dz=$$
$$=\frac{2}{l}\frac{\lambda_0}{\lambda_1}\int_0^l\psi(z)\sin\frac{n\pi}{l}z\,dz+\frac{b^*}{\lambda_1 l}\int_0^l\psi^2(z)\sin\frac{n\pi}{l}z\,dz.$$

Then, with consideration of [3], we obtain

$$A_n^* = \frac{M_n^* K_0\left(\frac{n\pi}{l} r_2\right) - N_n^* K_0\left(\frac{n\pi}{l} r_1\right)}{I_0\left(\frac{n\pi}{l} r_1\right) K_0\left(\frac{n\pi}{l} r_2\right) - I_0\left(\frac{n\pi}{l} r_2\right) K_0\left(\frac{n\pi}{l} r_1\right)},$$

$$B_n^* = \frac{N_n^* I_0\left(\frac{n\pi}{l} r_1\right) - M_n^* I_0\left(\frac{n\pi}{l} r_2\right)}{I_0\left(\frac{n\pi}{l} r_1\right) K_0\left(\frac{n\pi}{l} r_2\right) - I_0\left(\frac{n\pi}{l} r_2\right) K_0\left(\frac{n\pi}{l} r_1\right)}.$$

For C_n^* and D_n^*, considering (22), we obtain

$$C_n^* = \frac{\pi^2 \beta_n^2 J_0^2(\beta_n r_2)}{2\left[J_0^2(\beta_n r_1) - J_0^2(\beta_n r_2)\right]} \int_{r_1}^{r_2} r\xi(r)\left[\frac{\lambda_0}{\lambda_1} + \frac{b^*}{2\lambda_1}\xi(r)\right] V_0(\beta_n r)\, dr,$$

$$D_n^* = \frac{E_n^* - C_n^* \operatorname{ch} \beta_n l}{\operatorname{sh} \beta_n l},$$

where

$$E_n^* = \frac{\pi^2 \beta_n^2 J_0^2(\beta_n r_2)}{2\left[J_0^2(\beta_n r_1) - J_0^2(\beta_n r_2)\right]} \int_{r_1}^{r_2} r\zeta(r)\left[\frac{\lambda_0}{\lambda_1} + \frac{b^*}{2\lambda_1}\zeta(r)\right] V_0(\beta_n r)\, dr.$$

The analogous expressions can be found for other types of walls that participate in heat transfer.

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